ARCH 2615-5615

Steel beams

COMPRESSION

TENSION

σ = STRESS

b

d

C

T

²⁄₃d

σ = STRESS   MOMENT ARM
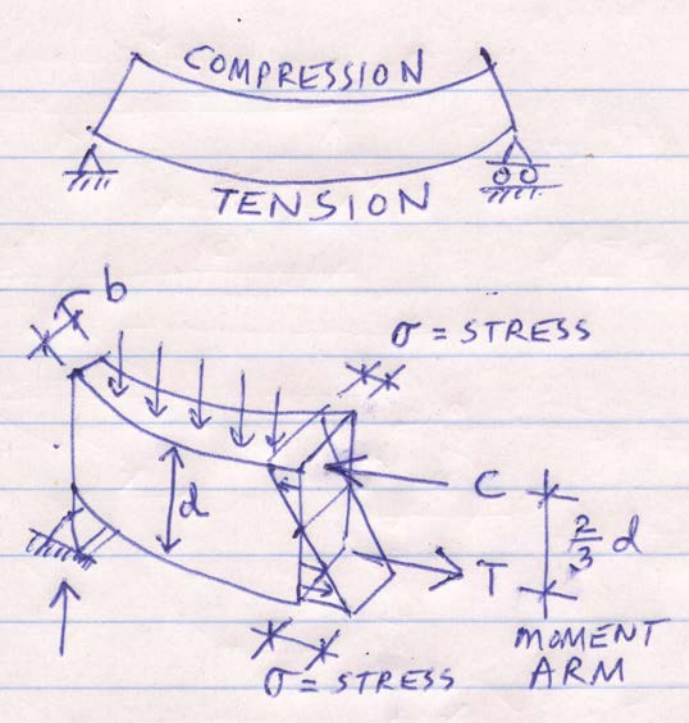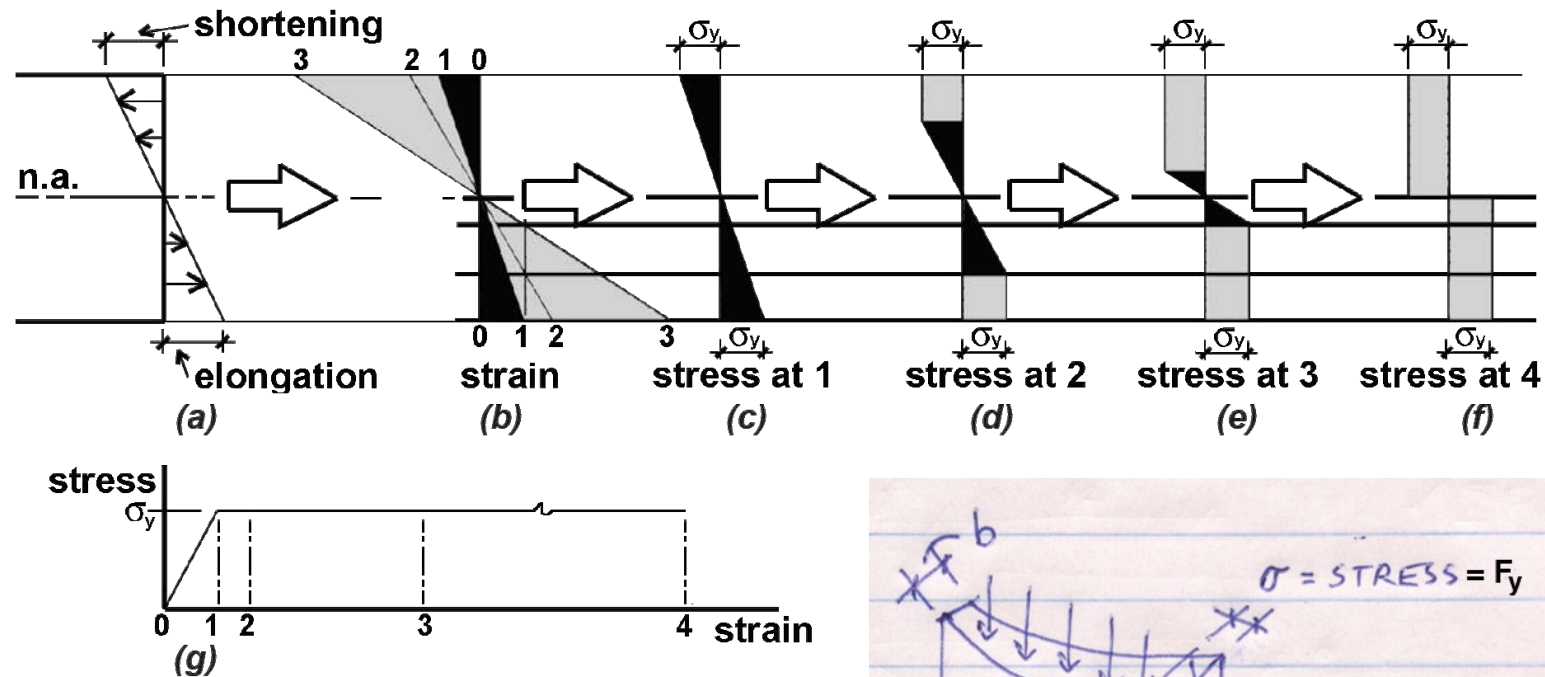
The force couple consisting of the compressive and tensile resultants is what we call the "bending moment" at the section where the free-body diagram is cut: $M = C(2/3)(d)$.

But, from horizontal equilibrium:
$C = T = ½(σ)(d/2)(b)$

Substituting C into the first equation:
$M = ½(σ)(d/2)(b)(2/3)(d) = (σ)(bd^2/6) = (σ)(S_x)$

S_x is called the "section modulus"

But in steel design, we take advantage of the additional strength beyond the elastic moment corresponding to the section modulus.
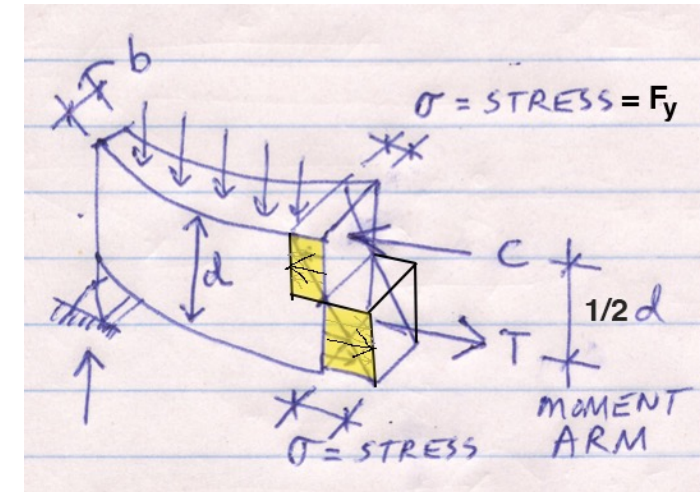

shortening
3   2 1 0
n.a.
elongation
(a)

0 1 2   3
strain
(b)

stress at 1
(c)

stress at 2
(d)

stress at 3
(e)

stress at 4
(f)

stress
σ_y
0 1 2   3   4 strain
(g)

The equilibrium equations change:
$M = C(1/2)(d)$.

From horizontal equilibrium:
$C = T = ½(F_y)(d/2)(b)$

Substituting C into the first equation:
$M = (F_y)(d/2)(b)(1/2)(d) = (F_y)(bd^2/4) = (F_y)(Z_x)$

Z_x is called the "plastic section modulus" and $Z_x = M / F_y$

b

σ = STRESS = F_y

d

C

T

1/2 d

σ = STRESS   MOMENT ARM

Compact sections and the beam design equation

The equation for plastic section modulus, $Z_x = M/F_y$, presumes that the cross section is able to reach a state of complete yielding before one of two types of buckling occurs: either (a) lateral-torsional buckling within any unbraced segment along the length of the span or (b) local flange or web buckling.
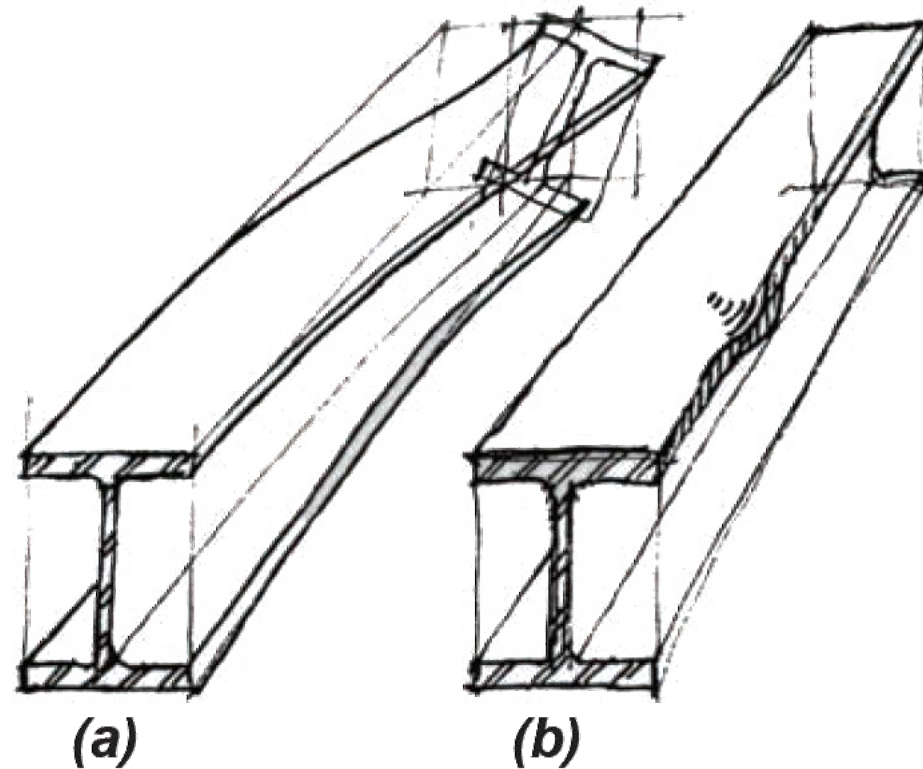
(a) (b)

Compact sections and the beam design equation

The equation for plastic section modulus, $Z_x = M/F_y$, presumes that the cross section is able to reach a state of complete yielding before one of two types of buckling occurs: either lateral-torsional buckling within any unbraced segment along the length of the span or local flange or web buckling.

Therefore, to use this equation in design, based on the maximum moment encountered, the beam must be protected from both of these buckling modes, in the first case by limiting the effective length (typically happens "automatically" since the compressive flange is "braced" by the floor deck) and, in the second case, by regulating the proportions of the beam flange and web (i.e., using a so-called **compact section**).

Then, rewriting this equation in the form most useful for steel design, by adding a safety factor that limits the maximum stress in the beam to $0.6F_y$, we get:

$$Z_{req} = M_{max} / (0.6F_y)$$

where M_{max} = the maximum bending moment (in-kips), F_y is the yield stress of the steel (ksi), and 0.6 is a safety factor for bending. The units of the required plastic section modulus are in^3.

From Table A-4.15, select lightest section with a plastic section modulus of at least 40.77 in^3

Select a W16x26

Now, check for shear and deflection:

Table A-4.15: Plastic section modulus (Z_x) values: lightest laterally braced steel compact shapes for bending, F_y = 50 ksi

Shape	Z_x (in^3)	^2L_p (ft)	Shape	Z_x (in^3)	^2L_p (ft)	Shape	Z_x (in^3)	^2L_p (ft)
W6 × 8.5[1]	5.59	3.14	W21 × 55	126	6.11	W40 × 211	906	8.87
W6 × 9[1]	6.23	3.20	W24 × 55	134	4.73	W40 × 215	964	12.5
W8 × 10[1]	8.77	3.14	W21 × 62	144	6.25	W44 × 230	1100	12.1
W10 × 12[1]	12.5	2.87	W24 × 62	153	4.87	W40 × 249	1120	12.5
W12 × 14	17.4	2.66	W21 × 68	160	6.36	W44 × 262	1270	12.3
W12 × 16	20.1	2.73	W24 × 68	177	6.61	W44 × 290	1410	12.3
W10 × 19	21.6	3.09	W24 × 76	200	6.78	W40 × 324	1460	12.6
W12 × 19	24.7	2.90	W24 × 84	224	6.89	W44 × 335	1620	12.3
W10 × 22	26.0	4.70	W27 × 84	244	7.31	W40 × 362	1640	12.7
W12 × 22	29.3	3.00	W30 × 90	283	7.38	W40 × 372	1680	12.7
W14 × 22	33.2	3.67	W30 × 99	312	7.42	W40 × 392	1710	9.33
W12 × 26	37.2	5.33	W30 × 108	346	7.59	W40 × 397	1800	12.9
W14 × 26	40.2	3.81	W30 × 116	378	7.74	W40 × 431	1960	12.9
W16 × 26	44.2	3.96	W33 × 118	415	8.19	W36 × 487	2130	14.0
W14 × 30	47.3	5.26	W33 × 130	467	8.44	W40 × 503	2320	13.1
W16 × 31	54.0	4.13	W36 × 135	509	8.41	W36 × 529	2330	14.1
W14 × 34	54.6	5.40	W33 × 141	514	8.58	W40 × 593	2760	13.4
W18 × 35	66.5	4.31	W40 × 149	598	8.09	W36 × 652	2910	14.5
W16 × 40	73.0	5.55	W36 × 160	624	8.83	W36 × 655	3080	13.6
W18 × 40	78.4	4.49	W40 × 167	693	8.48	W36 × 723	3270	14.7
W21 × 44	95.4	4.45	W36 × 182	718	9.01	W36 × 802	3660	14.9
W21 × 48	107	6.09	W40 × 183	774	8.80	W36 × 853	3920	15.1
W21 × 50	110	4.59	W40 × 199	869	12.2	W36 × 925	4130	15.0
W18 × 55	112	5.90						

For steel wide-flange shapes, simplified procedures have been developed, based on the **average** stress on the cross section, neglecting the overhanging flange areas; that is:

$$\tau_{max} = \frac{V}{dt_w} = V / A_w$$

(where τ_{max} = the maximum shear stress within the cross section, V = the total shear force at the cross section, d = the cross-sectional depth, and t_w = the web thickness.

Cross section   Shear stress
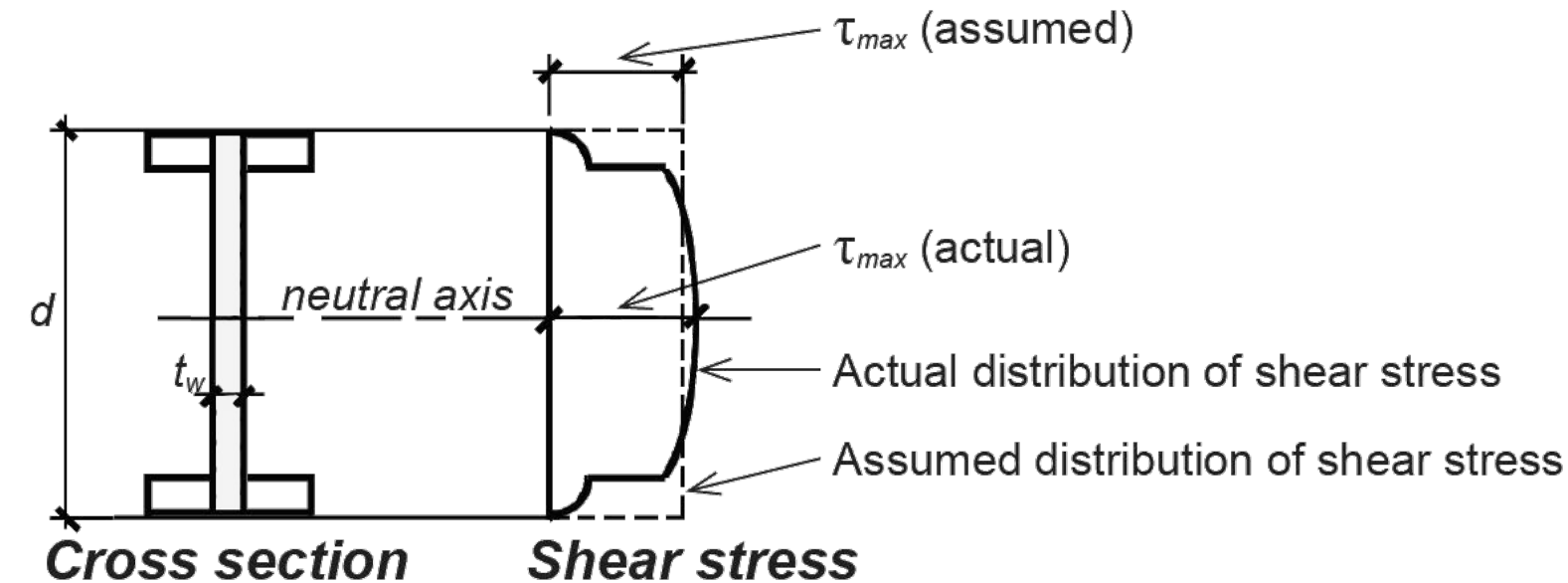
The "allowable" shear stress depends on the "slenderness" of the cross section (see Table A-4.3) and is set at $0.4F_y$ or $0.36F_y$ so that the equation for checking shear becomes:

Required web area, $A_w = V / (0.4F_y)$ or $A_w = V / (0.36F_y)$

We find section properties for the **W16x26** beam in Table A-4.3.

This is also where we find out whether to use a safety factor of 0.4 or 0.36.

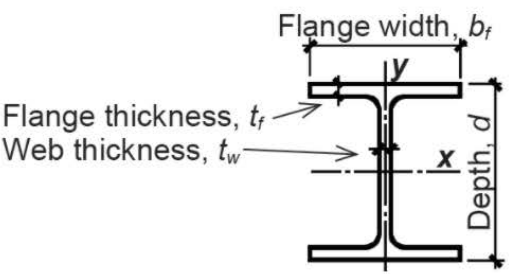From footnote 3 marked next to the section, we use a safety factor of 3.6.

Since V = 14.56 k and F_y = 50 ksi, the required web area, A_w = 14.56 / (0.36 x 50) = **0.809 in²**

We compare this required web area to the actual web area by finding d and t_w in **Table A-4.3.**

d = 15.7 in. and t_w = 0.25 in. Therefore, the actual web area = 15.7 x 0.25 = **3.925 in²**.

Since the actual web area is greater or equal to the required web area, ==**the section is OK for shear.**==

Table A-4.3: Dimensions and properties of steel W sections[5]

Flange width, b_f
Flange thickness, t_f
Web thickness, t_w
Depth, d

Cross-sectional area = A
Moment of inertia = I
Section modulus, $S_x = 2I_x/d$
Sectional modulus, $S_y = 2I_x/b_f$
Radius of gyration, $r_x = \sqrt{I_x/A}$
Radius of gyration, $r_y = \sqrt{I_y/A}$

Designa-tion	A (in²)	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	S_x (in³)	Z_x (in³)	I_x (in⁴)	I_y (in⁴)	r_y (in.)
W14 × 426	125	18.7	1.88	16.7	3.04	706	869	6600	2360	4.34
W14 × 455	134	19.0	2.02	16.8	3.21	756	936	7190	2560	4.38
W14 × 500	147	19.6	2.19	17.0	3.50	838	1050	8210	2880	4.43
W14 × 550	162	20.2	2.38	17.2	3.82	931	1180	9430	3250	4.49
W14 × 605	178	20.9	2.60	17.4	4.16	1040	1320	10800	3680	4.55
W14 × 665	196	21.6	2.83	17.7	4.52	1150	1480	12400	4170	4.62
W14 × 730	215	22.4	3.07	17.9	4.91	1280	1660	14300	4720	4.69
W14 × 808	238	22.8	3.74	18.6	5.12	1390	1830	15900	5550	4.83
W14 × 873	257	23.6	3.94	18.8	5.51	1530	2030	18100	6170	4.90
W16 × 26[3,4]	7.68	15.7	0.250	5.50	0.345	38.4	44.2	301	9.59	1.12
W16 × 31[4]	9.13	15.9	0.275	5.53	0.440	47.2	54.0	375	12.4	1.17
W16 × 36[4]	10.6	15.9	0.295	6.99	0.430	56.5	64.0	448	24.5	1.52
W16 × 40[4]	11.8	16.0	0.305	7.00	0.505	64.7	73.0	518	28.9	1.57
W16 × 45[4]	13.3	16.1	0.345	7.04	0.565	72.7	82.3	586	32.8	1.57
W16 × 50[4]	14.7	16.3	0.380	7.07	0.630	81.0	92.0	659	37.2	1.59
W16 × 57	16.8	16.4	0.430	7.12	0.715	92.2	105	758	43.1	1.60

Notes:

1. Section not compact for steel with F_y = 36 ksi or F_y = 50 ksi.

2. Section compact for steel with F_y = 36 ksi, but not compact for steel with F_y = 50 ksi.

3. Section webs do not meet slenderness criteria for shear for which the allowable stress can be taken as $F_v = 0.4F_y$; instead, use a reduced allowable shear stress, $F_v = 0.36F_y$.

4. Section is slender for compression with F_y = 50 ksi.

5. W-shapes are grouped together with common inner roller dimensions (i.e., web "lengths" excluding fillets)

The last step in the beam design process is to **check the W16x26 for deflection.**

Here, the relevant parameters are material properties (modulus of elasticity, E), sectional properties (moment of inertia, I_x), span (L), and load (w).
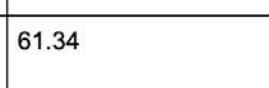
Rather than using the specialized equation for maximum mid-span deflection of a uniformly-loaded simply-supported beams, which is:
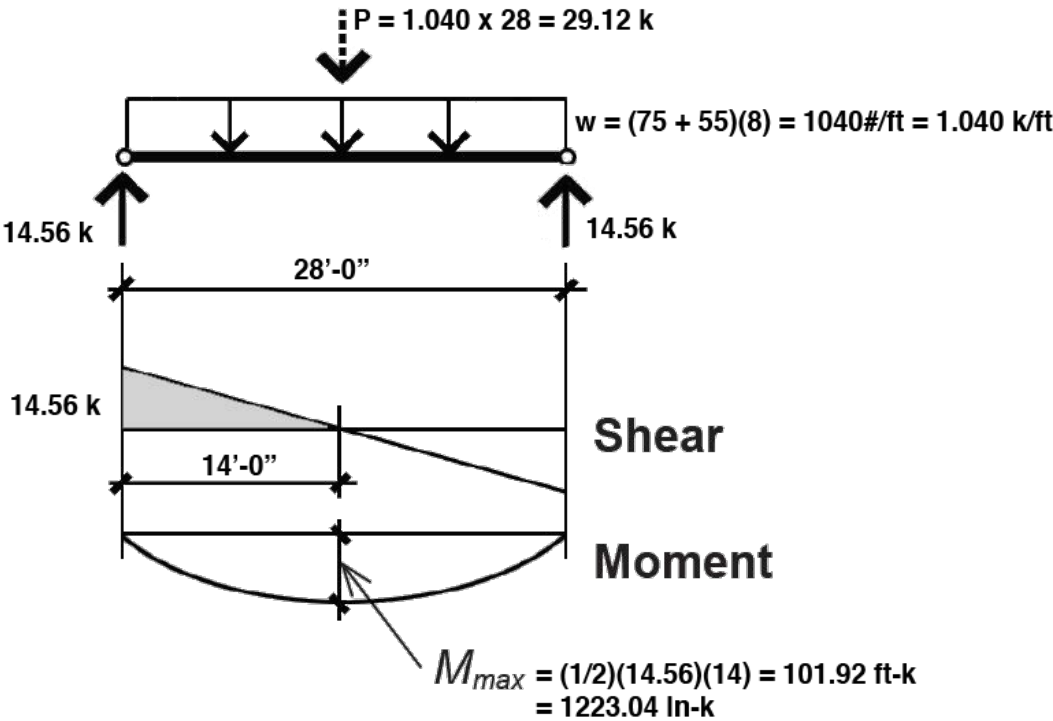
$\Delta = 5wL^4 / (384EI_x)$

We'll use the equation from the more flexible Appendix Table A-4.17 (same as A-3.15):

$\Delta = CP(L/12)^3 / (EI_x)$ where the coefficient, C is found from the table: **C = 22.46** in this case.

Table A-3.15: Maximum (actual) deflection in a beam[1,2,3]

Deflection coefficient, C, for maximum (actual) deflection, Δ (in.), where $\Delta = \dfrac{CP(L/12)^3}{EI_x}$				
	⊿———⊿	⊿———⊿	⊿———⊿	———⊿
$P = w(L/12)$	22.46	9.33	4.49	216
↓P	35.94	16.07	8.99	n/a
↓P ↓P	61.34	26.27	13.31	n/a
↓P↓P↓P	85.54	36.12	17.97	n/a
↓P	n/a	n/a	n/a	576

P = 1.040 x 28 = 29.12 k

w = (75 + 55)(8) = 1040#/ft = 1.040 k/ft

14.56 k   14.56 k

28'-0"

14.56 k

14'-0"

Shear

Moment

M_{max} = (1/2)(14.56)(14) = 101.92 ft-k
= 1223.04 In-k

The other parameters are easily determined:

$P = w(L/12)$ where w is either the live load or the total load (#/ft) and L is the span in inches (so $L/12$ is the span in feet).

Now, it turns out that we need to check the maximum deflection under two load scenarios: total load and live load. **Why??**

Because live load by itself might crack a "plaster" ceiling; while total load deflection might be unsightly, or correspond to vibration or bounciness in the floor.

Start with total load deflection.

Chapter 4 — Steel: Appendix

Table A-4.1: Steel properties[1]

Category	ASTM designation	Yield stress, F_y (ksi)	(Ultimate) tensile stress, F_u (ksi)	Preferred for these shapes
Carbon	A36 A500 Gr. B A500 Gr. B A53 Gr. B	36 42 46 [2]35	58 58 58 60	M, S, C, MC, L, plates[4] and bars HSS round[5] HSS rectangular[5] Pipe
High-strength, low-alloy	A992 A572 Gr. 50	50 50	65 65	[3]W HP

Notes:

1. **The modulus of elasticity, E, for these steels can be taken as 29,000 ksi.**

Table A-4.3: Dimensions and properties of steel W sections[5]

Flange width, b_f
Flange thickness, t_f
Web thickness, t_w
Depth, d

Cross-sectional area = A
Moment of inertia = I
Section modulus, $S_x = 2I_x/d$
Sectional modulus, $S_y = 2I_x/b_f$
Radius of gyration, $r_x = \sqrt{I_x/A}$
Radius of gyration, $r_y = \sqrt{I_y/A}$

Designa-tion	A (in^2)	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	S_x (in^3)	Z_x (in^3)	I_x (in^4)	I_y (in^4)	r_y (in.)
W14 × 730	215	22.4	3.07	17.9	4.91	1280	1660	14300	4720	4.69
W14 × 808	238	22.8	3.74	18.6	5.12	1390	1830	15900	5550	4.83
W14 × 873	257	23.6	3.94	18.8	5.51	1530	2030	18100	6170	4.90
W16 × 26[3,4]	7.68	15.7	0.250	5.50	0.345	38.4	44.2	301	9.59	1.12
W16 × 31[4]	9.13	15.9	0.275	5.53	0.440	47.2	54.0	375	12.4	1.17

TOTAL LOAD DEFLECTION:

So, we can now compute the *actual* total load deflection:
$\Delta = CP(L/12)^3 / (EI_x)$, or
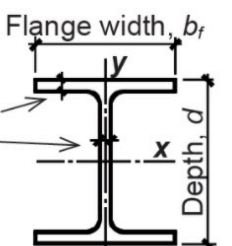$\Delta = (22.46)(29.12)(28)^3 / (29,000 \times 301) = $ **1.64 in.**

The *allowable* total load deflection can be found in the footnotes to Table A-4.17 (or A-3.15):

For total loads (combined live and dead), the typical basic floor beam limit is **L/240** while typical roof beam limits are $L/120$, $L/180$, or $L/240$ (for no ceiling, nonplaster ceiling, or plaster ceiling respectively).

Using the typical limit of L/240 (with L expressed in inches), we get an allowable value of 28 x 12 / 240 = **1.4 in.**

Conclusion: Since the actual total-load deflection is greater than the allowable total-load deflection, the W16x26 is NOT OK for total-load deflection!
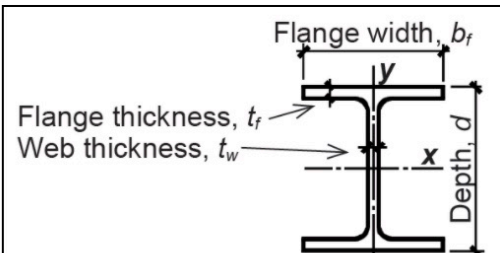
Chapter 4 — Steel: Appendix

Table A-4.1: Steel properties[1]

Category	ASTM designation	Yield stress, F_y (ksi)	(Ultimate) tensile stress, F_u (ksi)	Preferred for these shapes
Carbon	A36 A500 Gr. B A500 Gr. B A53 Gr. B	36 42 46 [2]35	58 58 58 60	M, S, C, MC, L, plates[4] and bars HSS round[5] HSS rectangular[5] Pipe
High-strength, low-alloy	A992 A572 Gr. 50	50 50	65 65	[3]W HP

Notes:

1. **The modulus of elasticity, E, for these steels can be taken as 29,000 ksi.**

Table A-4.3: Dimensions and properties of steel W sections[5]

Flange width, b_f

Flange thickness, t_f
Web thickness, t_w

Depth, d

Cross-sectional area = A
Moment of inertia = I
Section modulus, $S_x = 2I_x/d$
Sectional modulus, $S_y = 2I_x/b_f$
Radius of gyration, $r_x = \sqrt{I_x/A}$
Radius of gyration, $r_y = \sqrt{I_y/A}$

Designation	A (in²)	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	S_x (in³)	Z_x (in³)	I_x (in⁴)	I_y (in⁴)	r_y (in.)
W14 × 730	215	22.4	3.07	17.9	4.91	1280	1660	14300	4720	4.69
W14 × 808	238	22.8	3.74	18.6	5.12	1390	1830	15900	5550	4.83
W14 × 873	257	23.6	3.94	18.8	5.51	1530	2030	18100	6170	4.90
W16 × 26[3,4]	7.68	15.7	0.250	5.50	0.345	38.4	44.2	301	9.59	1.12
W16 × 31[4]	9.13	15.9	0.275	5.53	0.440	47.2	54.0	375	12.4	1.17

LIVE LOAD DEFLECTION…
is the same except with a different load in the equation.
The live load, w = 75 x 8 = 600#/ft = 0.6 k/ft, so…
The live load, P = 0.6 x 28 = 16.8 k

The *actual* live load deflection is:
$\Delta = CP(L/12)^3 / (EI_x)$, or
$\Delta = (22.46)(16.8)(28)^3 / (29,000 \times 301) = \textbf{0.95 in.}$

The *allowable* live load deflection can be found in the footnotes to Table A-4.17 (or A-3.15):

For live loads only, the typical basic floor beam limit is **L/360** while typical roof beam limits are $L/180$, $L/240$, or $L/360$ (for no ceiling, nonplaster ceiling, or plaster ceiling respectively).

Using the typical limit of L/360 (with L expressed in inches), we get an allowable value of 28 x 12 / 360 = **0.93 in.**

Conclusion: Since the actual total-load deflection is greater than the allowable total-load deflection, the W16x26 is not OK for live-load deflection!

To improve the deflection performance of the beam, find a cross section with a larger moment of inertia (and an acceptable plastic section modulus for bending stress).

But we will not redesign this beam: only note that it is not OK for live load deflection (but it's close).
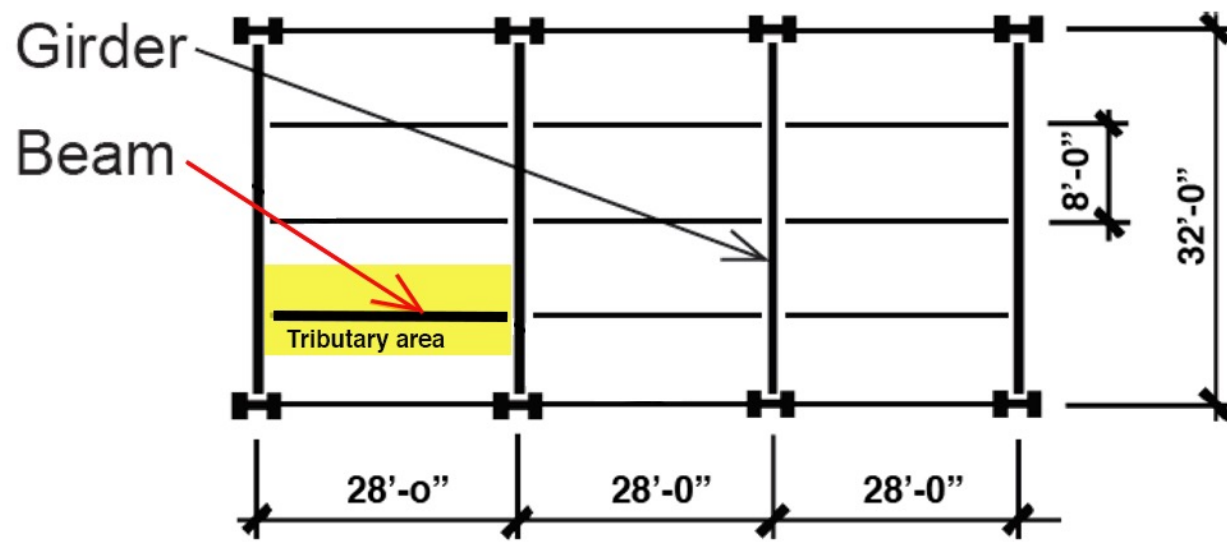
Framing plan

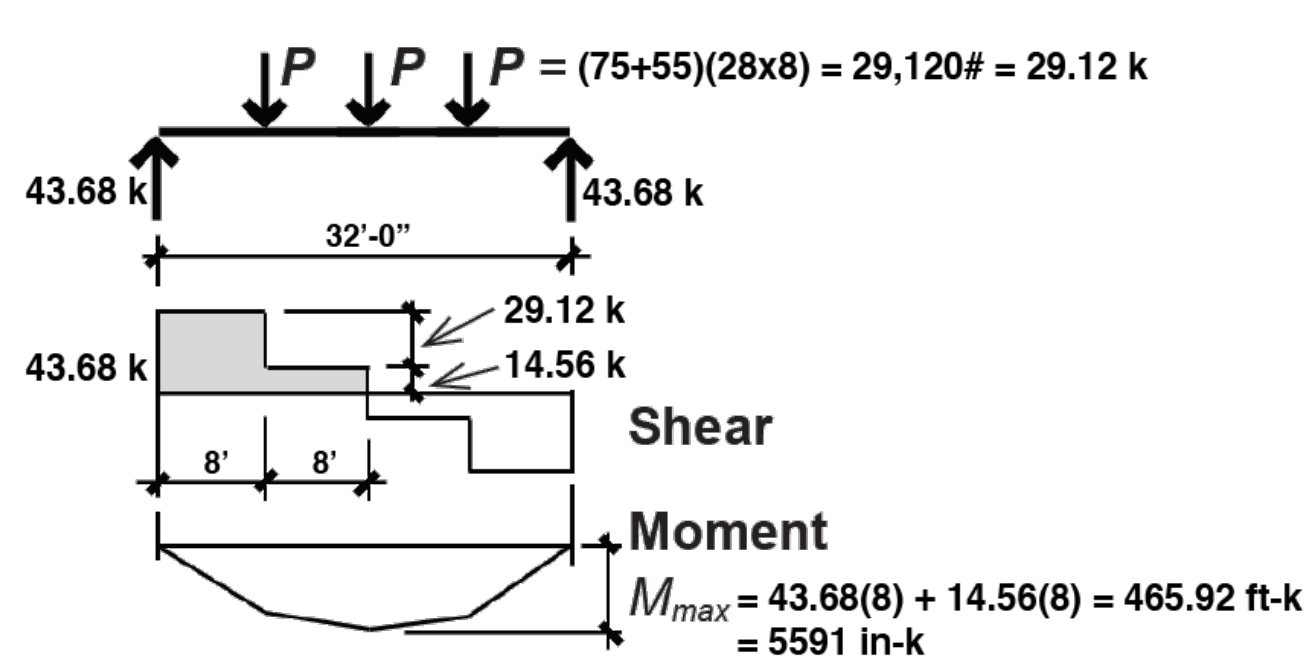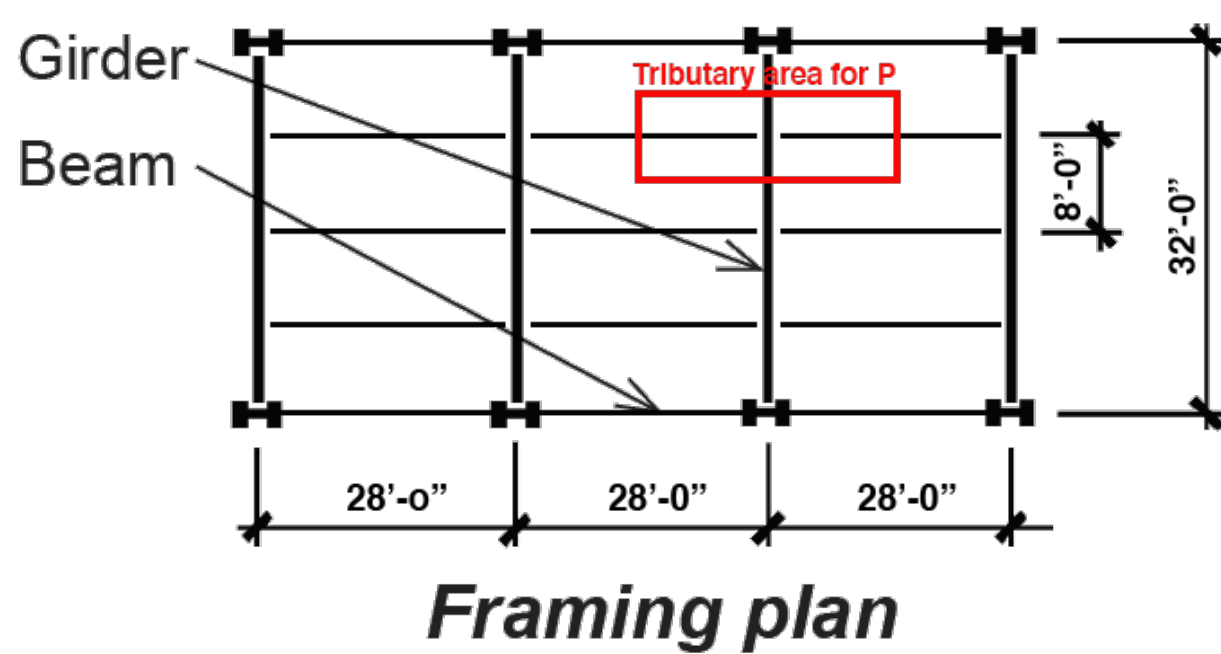Design typical beam (no live load reduction).
Assume L = 75 psf and D = 55 psf
Use A-992 steel with F_y = 50 ksi

Just for the record, if we were going to account for live load reduction, we would use a tributary area = 28 x 8 = 224 ft² and a live load element factor, K_{LL} = 2.

The reduced live load would therefore be 75 x [0.25 + 15 / sqrt(2 x 224)] = 75 x 0.96 = 71.9 psf

But, for this example, we used the unreduced live load, L = 75 psf...

Now, on to girder design, also using the unreduced live load.

Girder

Beam

Tributary area for P

8'-0"

32'-0"

28'-0"   28'-0"   28'-0"

Framing plan

P P P = (75+55)(28x8) = 29,120# = 29.12 k

43.68 k   43.68 k

32'-0"

43.68 k   29.12 k
             14.56 k

Shear

8'   8'

Moment

M_{max} = 43.68(8) + 14.56(8) = 465.92 ft-k
= 5591 in-k

Design typical girder (no live load reduction).
Assume L = 75 psf and D = 55 psf
Use A-992 steel with F_y = 50 ksi

Chapter 4 — Steel: Appendix

Table A-4.1: Steel properties[1]

Category	ASTM designation	Yield stress, F_y (ksi)	(Ultimate) tensile stress, F_u (ksi)	Preferred for these shapes
Carbon	A36	36	58	M, S, C, MC, L, plates[4] and bars
	A500 Gr. B	42	58	HSS round[5]
	A500 Gr. B	46	58	HSS rectangular[5]
	A53 Gr. B	[2]35	60	Pipe
High-strength, low-alloy	A992	50	65	[3]W
	A572 Gr. 50	50	65	HP

$Z_{req} = M_{max} / (0.6F_y)$

Z_{req} = 5591 / (0.6 x 50) = **186.4 in³**

Select provisional section from Table A-4.15

From Table A-4.15, select lightest section with a plastic section modulus of at least 186.4 in^3

Select a W24x76

Now, check for shear and deflection:

Table A-4.15: Plastic section modulus (Z_x) values: lightest laterally braced steel compact shapes for bending, F_y = 50 ksi

Shape	Z_x (in^3)	^2L_p (ft)	Shape	Z_x (in^3)	^2L_p (ft)	Shape	Z_x (in^3)	^2L_p (ft)
W6 × 8.5[1]	5.59	3.14	W21 × 55	126	6.11	W40 × 211	906	8.87
W6 × 9[1]	6.23	3.20	W24 × 55	134	4.73	W40 × 215	964	12.5
W8 × 10[1]	8.77	3.14	W21 × 62	144	6.25	W44 × 230	1100	12.1
W10 × 12[1]	12.5	2.87	W24 × 62	153	4.87	W40 × 249	1120	12.5
W12 × 14	17.4	2.66	W21 × 68	160	6.36	W44 × 262	1270	12.3
W12 × 16	20.1	2.73	W24 × 68	177	6.61	W44 × 290	1410	12.3
W10 × 19	21.6	3.09	W24 × 76	200	6.78	W40 × 324	1460	12.6
W12 × 19	24.7	2.90	W24 × 84	224	6.89	W44 × 335	1620	12.3
W10 × 22	26.0	4.70	W27 × 84	244	7.31	W40 × 362	1640	12.7
W12 × 22	29.3	3.00	W30 × 90	283	7.38	W40 × 372	1680	12.7
W14 × 22	33.2	3.67	W30 × 99	312	7.42	W40 × 392	1710	9.33
W12 × 26	37.2	5.33	W30 × 108	346	7.59	W40 × 397	1800	12.9
W14 × 26	40.2	3.81	W30 × 116	378	7.74	W40 × 431	1960	12.9
W16 × 26	44.2	3.96	W33 × 118	415	8.19	W36 × 487	2130	14.0
W14 × 30	47.3	5.26	W33 × 130	467	8.44	W40 × 503	2320	13.1
W16 × 31	54.0	4.13	W36 × 135	509	8.41	W36 × 529	2330	14.1
W14 × 34	54.6	5.40	W33 × 141	514	8.58	W40 × 593	2760	13.4
W18 × 35	66.5	4.31	W40 × 149	598	8.09	W36 × 652	2910	14.5
W16 × 40	73.0	5.55	W36 × 160	624	8.83	W36 × 655	3080	13.6
W18 × 40	78.4	4.49	W40 × 167	693	8.48	W36 × 723	3270	14.7
W21 × 44	95.4	4.45	W36 × 182	718	9.01	W36 × 802	3660	14.9
W21 × 48	107	6.09	W40 × 183	774	8.80	W36 × 853	3920	15.1
W21 × 50	110	4.59	W40 × 199	869	12.2	W36 × 925	4130	15.0
W18 × 55	112	5.90						

We find section properties for the **W24x76** beam in Table A-4.3.

This is also where we find out whether to use a safety factor of 0.4 or 0.36.

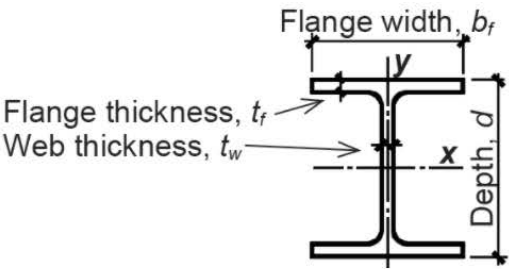Without footnote 3 marked next to the section, we use a safety factor of 4.0.

Since V = 43.68 k and F_y = 50 ksi, the required web area, A_w = 43.68 / (0.40 x 50) = **2.184 in²**

We compare this required web area to the actual web area by finding d and t_w in **Table A-4.3.**

d = 23.9 in. and t_w = 0.44 in. Therefore, the actual web area = 23.9 x 0.44 = **10.52 in².**

Since the actual web area is greater or equal to the required web area, **the section is OK for shear.**

Table A-4.3: Dimensions and properties of steel W sections[5]

Flange width, b_f
Flange thickness, t_f
Web thickness, t_w
Depth, d

Cross-sectional area = A
Moment of inertia = I
Section modulus, $S_x = 2I_x/d$
Sectional modulus, $S_y = 2I_x/b_f$
Radius of gyration, $r_x = \sqrt{I_x/A}$
Radius of gyration, $r_y = \sqrt{I_y/A}$

Designa-tion	A (in²)	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	S_x (in³)	Z_x (in³)	I_x (in⁴)	I_y (in⁴)	r_y (in.)
W24 × 68[4]	20.1	23.7	0.415	8.97	0.585	154	177	1830	70.4	1.87
W24 × 76[4]	22.4	23.9	0.440	8.99	0.680	176	200	2100	82.5	1.92
W24 × 84[4]	24.7	24.1	0.470	9.02	0.770	196	224	2370	94.4	1.95
W24 × 94[4]	27.7	24.3	0.515	9.07	0.875	222	254	2700	109	1.98
W24 × 103[4]	30.3	24.5	0.550	9.00	0.980	245	280	3000	119	1.99

Notes:

1. Section not compact for steel with F_y = 36 ksi or F_y = 50 ksi.

2. Section compact for steel with F_y = 36 ksi, but not compact for steel with F_y = 50 ksi.

3. Section webs do not meet slenderness criteria for shear for which the allowable stress can be taken as $F_v = 0.4F_y$; instead, use a reduced allowable shear stress, $F_v = 0.36F_y$.

4. Section is slender for compression with F_y = 50 ksi.

5. W-shapes are grouped together with common inner roller dimensions (i.e., web "lengths" excluding fillets)

The last step in the beam design process is to **check the W24x76 for deflection.**

Here, the relevant parameters are material properties (modulus of elasticity, E), sectional properties (moment of inertia, I_x), span (L), and load (w).
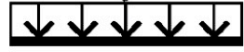
Rather than using the specialized equation for maximum mid-span deflection of a uniformly-loaded simply-supported beams, which is:
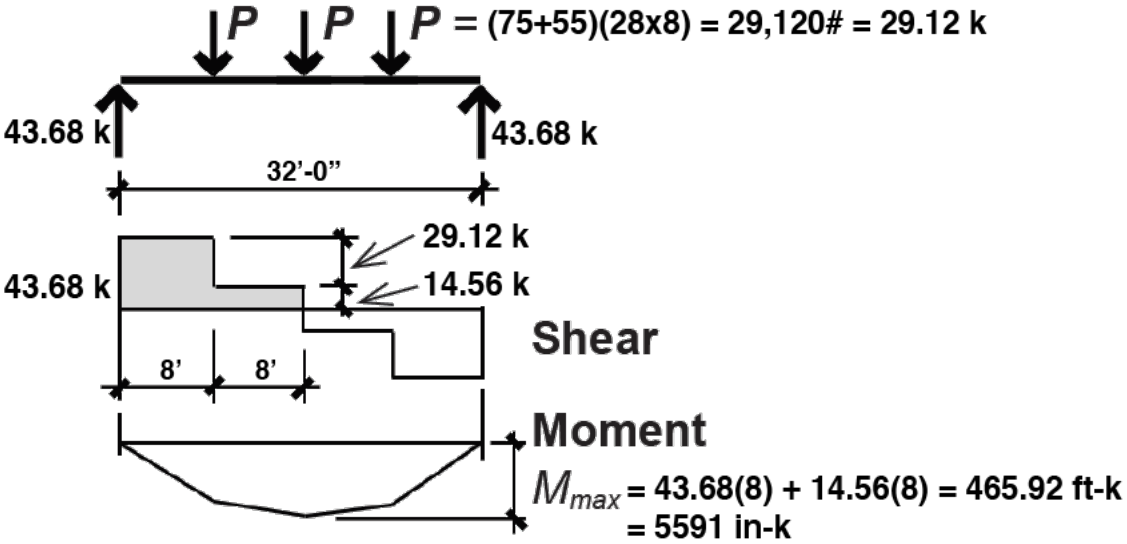
$\Delta = 5wL^4 / (384EI_x)$

We'll use the equation from the more flexible Appendix Table A-4.17 (same as A-3.15):

$\Delta = CP(L/12)^3 / (EI_x)$ where the coefficient, C is found from the table: **$C = 85.54$** in this case.

$P \quad P \quad P = (75+55)(28 \times 8) = 29,120\# = 29.12 \text{ k}$

43.68 k | 43.68 k

32'-0"

29.12 k
14.56 k

Shear

8' | 8'

Moment

$M_{max} = 43.68(8) + 14.56(8) = 465.92 \text{ ft-k}$
$= 5591 \text{ in-k}$

Table A-4.17: Maximum (actual) deflection in a beam[1,2,3]

Deflection coefficient, C, for maximum (actual) deflection, Δ (in.), where $\Delta = \dfrac{CP(L/12)^3}{EI_x}$				
$P = w(L/12)$	22.46	9.33	4.49	216
P	35.94	16.07	8.99	n/a
$P \quad P$	61.34	26.27	13.31	n/a
$P \quad P \quad P$	85.54	36.12	17.97	n/a
P	n/a	n/a	n/a	576

The other parameters are easily determined:

$P =$ **29.12 k for total load deflection (from diagram); and**
$P = (75)(28 \times 8) = 16,800\# =$ **16.8 k for live load deflection**

Start with total load deflection.
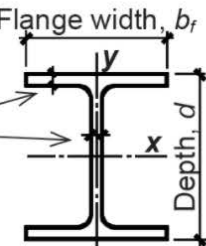
Chapter 4 — Steel: Appendix

Table A-4.1: Steel properties[1]

Category	ASTM designation	Yield stress, F_y (ksi)	(Ultimate) tensile stress, F_u (ksi)	Preferred for these shapes
Carbon	A36 A500 Gr. B A500 Gr. B A53 Gr. B	36 42 46 [2]35	58 58 58 60	M, S, C, MC, L, plates[4] and bars HSS round[5] HSS rectangular[5] Pipe
High-strength, low-alloy	A992 A572 Gr. 50	50 50	65 65	[3]W HP

Notes:

1. **The modulus of elasticity, E, for these steels can be taken as 29,000 ksi.**

Table A-4.3: Dimensions and properties of steel W sections[5]

Flange width, b_f

Flange thickness, t_f
Web thickness, t_w

Depth, d

Cross-sectional area = A
Moment of inertia = I
Section modulus, $S_x = 2I_x/d$
Sectional modulus, $S_y = 2I_x/b_f$
Radius of gyration, $r_x = \sqrt{I_x/A}$
Radius of gyration, $r_y = \sqrt{I_y/A}$

Designa-tion	A (in²)	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	S_x (in³)	Z_x (in³)	I_x (in⁴)	I_y (in⁴)	r_y (in.)
W24 × 68[4]	20.1	23.7	0.415	8.97	0.585	154	177	1830	70.4	1.87
W24 × 76[4]	22.4	23.9	0.440	8.99	0.680	176	200	2100	82.5	1.92
W24 × 84[4]	24.7	24.1	0.470	9.02	0.770	196	224	2370	94.4	1.95
W24 × 94[4]	27.7	24.3	0.515	9.07	0.875	222	254	2700	109	1.98
W24 × 103[4]	30.3	24.5	0.550	9.00	0.980	245	280	3000	119	1.99

TOTAL LOAD DEFLECTION:

So, we can now compute the *actual* total load deflection:
$\Delta = CP(L/12)^3 / (EI_x)$, or
$\Delta = (85.54)(29.12)(32)^3 / (29{,}000 \times 2100) = $ **1.34 in.**

The *allowable* total load deflection can be found in the footnotes to Table A-4.17 (or A-3.15):

For total loads (combined live and dead), the typical basic floor beam limit is **L/240** while typical roof beam limits are $L/120$, $L/180$, or $L/240$ (for no ceiling, nonplaster ceiling, or plaster ceiling respectively).

Using the typical limit of L/240 (with L expressed in inches), we get an allowable value of $32 \times 12 / 240 = $ **1.6 in.**

Conclusion: Since the actual total-load deflection is less than or equal to the allowable total-load deflection, the W24x76 is OK for total-load deflection!
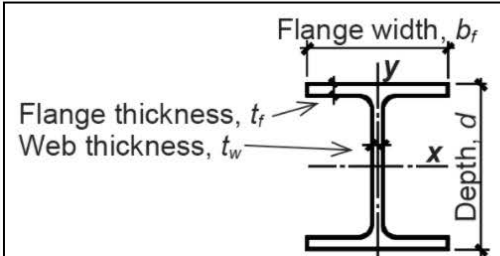
Chapter 4 — Steel: Appendix

Table A-4.1: Steel properties[1]

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High-strength, low-alloy	A992 A572 Gr. 50	50 50	65 65	[3]W HP

Notes:

1. **The modulus of elasticity, E, for these steels can be taken as 29,000 ksi.**

Table A-4.3: Dimensions and properties of steel W sections[5]

Flange width, b_f
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Cross-sectional area = A
Moment of inertia = I
Section modulus, $S_x = 2I_x/d$
Sectional modulus, $S_y = 2I_x/b_f$
Radius of gyration, $r_x = \sqrt{I_x/A}$
Radius of gyration, $r_y = \sqrt{I_y/A}$

Designation	A (in²)	d (in.)	t_w (in.)	b_f (in.)	t_f (in.)	S_x (in³)	Z_x (in³)	I_x (in⁴)	I_y (in⁴)	r_y (in.)
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W24 × 103[4]	30.3	24.5	0.550	9.00	0.980	245	280	3000	119	1.99

LIVE LOAD DEFLECTION:

So, we can now compute the *actual* live load deflection:
$\Delta = CP(L/12)^3 / (EI_x)$, or
$\Delta = (85.54)(16.8)(32)^3 / (29{,}000 \times 2100) = $ **0.77 in.**

The *allowable* total load deflection can be found in the footnotes to Table A-4.17 (or A-3.15):

For total loads (combined live and dead), the typical basic floor beam limit is **L/360** while typical roof beam limits are $L/180$, $L/240$, or $L/360$ (for no ceiling, nonplaster ceiling, or plaster ceiling respectively).

Using the typical limit of L/360 (with L expressed in inches), we get an allowable value of 32 x 12 / 360 = **1.07 in.**

Conclusion: Since the actual live-load deflection is less than or equal to the allowable live-load deflection, the W24x76 is OK for live-load deflection!

Conclusion: The W24x76 is good for bending, shear, and deflection, so it works!